Ovoids in $PG(3, q)$ and algebraic codes

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Abstract: An ovoid in $PG(3, K)$, K a skew field, is an Incidence-Geometric analogue of a sphere in the Euclidean 3-space. Apart from the elliptic quadrics (that is, nondegenerate quadrics in $PG(3, K)$ containing no lines, which exist for all fields K), the only other finite ovoids known, and projectively nonequivalent to elliptic ovoids, are the Tits ovoids which exist only in $PG(3, 2^t)$, for all $t > 4$ and odd. Its stabilizer in $PGL(4, 2^t)$ was discovered by Michio Suzuki (1962) as the final step in the (mammoth!) classification of finite 2-transitive groups with no nontrivial element fixing more than two symbols (that is, the so called Zassenhaus groups); and the ovoid itself was discovered by Jacques Tits (1962) as the set of absolute points of a polarity of the generalized 4-gon $W(2^t)$, the spherical building for the simple group $PSp(4, 2^t)$. These ovoids are also finite Moufang sets (that is, rank 1-spherical buildings as interpreted by Tits).

In this talk, I outline a few important facts about these ovoids and their stabilizers; describe their connections with inversive planes, Tits (q, q^2)-generalized 4-gon and translation planes. I also try to indicate the role of algebraic codes in these matters.